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SYSTEMATIZATION OF DIMENSIONLESS QUANTITIES BY GROUP THEORY

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Abstract—It is known that the physical quantities form, in algebraic sense, an infinite free abelian group. It is shown in this paper, that the dimensionless quantities of a given system form a finite free abelian group. It follows from this statement, that any element of the group may be obtained in the form of a whole exponent power product. The members of this power products are called basic elements. A new logical systematization of the dimensionless quantities by the group theory was possible.

The main results of this systematization are as follows:

1. The number of basic elements is identical with the degrees of freedom.
2. Any arbitrary proceeding in determination of the basic dimensionless quantities is eliminated, as the determination of the question, how many basic criteria there are and which may be these, is made according to exact instructions.
3. It is possible to explain the relations among the dimensionless quantities, namely it can be determined, that how many dimensionless quantities are included into the relation and which are these quantities.

Examples are given in the determination of the criteria of scaling-up. The basic equations in chemical engineering are discussed.

NOMENCLATURE

a, $\frac{\lambda}{\rho c_p}$, heat diffusivity (m²/h);
c_i, molar concentration of component i (mol/m³);
c_p, specific heat (kcal/kg degree);
d, characteristic length (or diameter) (m);
D, diffusivity (m²/h);
f', friction factor;
F, degree of freedom;
ΔH, molar heat of reaction (kcal/mol);
p, pressure (kg/m s²);
p, number of basic criteria, equations (5) and (6);
r, rate of reaction (mol/m³ h);
$r' = \frac{\nu_i r}{c_i}$, rate of reaction referred to unit time (1/h);
T, temperature (grade);
t, time (h);
v, linear flow velocity (m/h);
a, exponent, equations (4), (5), (11) and (13);
α, heat-transfer coefficient (kcal/m²h degree);
b, exponent, equations (12) and (13);
β, component, (mass-transfer) coefficient (m/h);
γ, momentum transfer coefficient $= \frac{f' \rho v}{2}$ (kg/m² h);
λ, thermal conductivity (kcal/m h degree);
ν_i, stoichiometric coefficient;
γ, kinematic viscosity (m²/h);
η, dynamic viscosity (kg/m h);
ρ, density (kg/m³);
ω, interfacial area per unit volume (m²/m³).

THE use of dimensionless quantities is widespread in the engineering practice. Chemical engineering forms no exception. This is due on the one hand

to the fact that often rather intricate relations between several variables can simply be characterized numerically by a dimensionless quantity. Beyond this point of view, which is actually one of convenience only, the formation of dimensionless quantities, i.e. of the ratio of quantities with identical dimensions, essentially means measurement. It is known that ordinary measuring is a comparison: the quantity to be measured is compared with a conventional (standard) quantity, i.e. with the unit. When a dimensionless quantity is formed, the quantity to be measured is compared with a characteristic quantity of the system, having the same dimension, instead of comparing it with a conventional unit. The formation of dimensionless quantities is therefore called the introduction of *eigen measure*. For example, every cylindrical body is characterized by its length (l) referred to its diameter (d) as a unit, i.e.

$$\frac{l}{d} = \epsilon. \tag{1a}$$

Another well known example is the Reynolds number

$$Re = \frac{\rho v d}{\eta} = \frac{v d}{\nu} \tag{1b}$$

which represents the ratio of convective and conductive momentum streams. As it is known, it was essential to recognize that the transition from laminar into turbulent flow characteristics does not depend on the *absolute* values of the convection or conduction momentum streams themselves, but on their ratio, i.e. on the *Re* number. Thus the dimensionless quantity is a scale number.

The formal systematization of dimensionless quantities is done on the basis of their formation. According to this, there are:

1. Simple or simplex dimensionless quantities (1a).
2. Compound or complex dimensionless quantities (1b).
3. Dimensionless quantities of efficiency character.

The aim of this paper is to show the possibility of another systematization of dimensionless quantities, on the basis of the relations among the dimensionless quantities, furnishing therefore information on the correlations between them. This systematization is done by means of the algebraic group theory.

It has been proved by Fleischmann [1] that the physical quantities form an infinite, free abelian group. This means that the criteria required from a group as an algebraic structure are fulfilled by the physical quantities. Only three of these criteria will be dealt with here, which are absolutely necessary for the understanding of the following.

I. A group is a (non-empty) set, S, among the elements of which there exists a relation (usually written as a multiplication), and this orders unambiguously to the element pair A, B of set S, an element C of the set:

$$AB = C \tag{2}$$

C is called the product of A and B.

II. The multiplication conceived in this way is commutative in the case of physical quantities. Therefore

$$AB = BA. \tag{3}$$

The groups which fulfil the criterion (3) are called *commutative* or *abelian* groups.

III. It can be shown [2] that *any* element of the free abelian group may be obtained as a product of a finite number of whole exponent generators:

$$X = C_1^{a_1} C_2^{a_2} \dots C_q^{a_q} = \prod_{i=1}^{q} C_i^{a_i}. \tag{4}$$

Among the generators, there are preferred sets which contain the *maximum number of independent elements*, and consist, at the same time, of the *minimum number* of generators. These specific sets of generators are called *bases*. Out of the elements C_i in relation (4), p elements are selected, where $p \leqslant q$, and marking these selected elements by B_i, *any* element of the group may be obtained by these in the form of a power product, where the exponent is a positive or negative whole number, or zero:

$$X = B_1^{a_1} B_2^{a_2} \dots B_p^{a_p} = \prod_{i=1}^{p} B_i^{a_i}. \tag{5}$$

The word "free" in the term free abelian group means that the generators are independent of each other, and no equation—restriction—exists between these elements. It is a logical consequence of this and has also been proved in detail [3] that the *number* of basic components, p, is identical with the degree of freedom, F, i.e.

$$p = F = N - M \tag{6}$$

where N is the number of all the elements, and M the number of equations (restrictions).

The term "degree of freedom" is used in the sense as first conceived by Gilliland and Reed [4]. Thus it represents the number of those independent variables which can be "selected freely" by the engineers, and the system will be defined unambiguously by fixing their values. The remaining variables, the number of which is M, are defined by M relations valid in the system. It will be seen subsequently, that the group theoretical treatment presented here does not only furnish the number of the independent variables to be selected freely by means of the identity $p \equiv F$, but it can also be shown by this treatment *which* of the system variables may be chosen freely.

If in the infinite sphere of physical quantities we restrict our investigations to a *system* in physicochemical sense, i.e. to a set of elements limited by a wall of *finite dimensions*, the number of the elements (which in our case are physical quantities) will also be a finite number (N).

Thus the definition of the system is as follows: A system, has finite dimensions; is limited by a wall of definite properties; and (just in consequence of the first two restrictions) is a group which can be completely described by a finite number of quantities (N elements). It follows from the above that any system may be considered as a subgroup of the infinite free abelian group representing all the physical quantities. This can be understood as follows:

Should G be the infinite free abelian group containing *all* the physical quantities as elements. It can be seen directly that N is a part set of G. Apart from this, the following requirement is also met by N.

The product of any two elements of N is included into N. This means, that the set is *closed* as regards the multiplication operation. N being a group itself, it also has a basis. That abelian group has a basis is a theorem which can be proved [5].

The dimensionless quantities form a *finite, free abelian group*, since they are obtained as ratios of physical quantities, i.e. according to the Axiom I of group theory as a result of multiplication, and because their number is finite as related to a system.

This subgroup also must have a basis, and as shown by equation (6), the number of its basic elements gives the degree of freedom of the dimensionless system. Of course the basic elements are also dimensionless in this case. Up to now the dimensionless basis has been called a "complete set" in the theory of similitude [6]. It is new, however, in this context that the number of the elements forming the "complete set" represents also a degree of freedom [see equation (6)]. It follows from the conception of the degree of freedom [3] that if out of the total number, N, of the variables of the system those forming a basis are selected, the data required and sufficient for the *unambiguous* description of the dimensionless system are determined at the same time. These data are called basic quantities and in the special case of dimensionless quantities *basic criteria*. The practical importance of the problem rests on this statement. All the basic criteria together also define a system, which is called the *basic system*. This is a fictitious system, since it does not exist in the physical reality, but the actually existing system may be expressed by this reduced fictitious system. Thus the number of the variables describing the system has been diminished. This is also of great importance in engineering practice.

In addition to this, when controlling a system, or at scaling up, it is not the same *how many* parameters are to be considered and *which* of these parameters will be, but according to the above, the parameters should be the basic criteria.

To illustrate the above arguments, applications for the three characteristic quantities occurring in chemical engineering science will be given. These quantities are: the heat, the momentum and the masses of chemical components, the

Table 1

	convection / conduction	transfer / convection	source / convection
Component	$Pe' = \frac{vd}{D}$	$St' = \frac{\beta}{v}$	$DaI = \frac{v_i r d}{c_i v}$
Heat	$Pe = \frac{vd}{a}$	$St = \frac{\alpha}{\rho c_p v}$	$DaII = \frac{\Delta H r d}{\rho c_p \Delta T v}$
Momentum	$Pe = \frac{vd}{\nu}$	$\frac{f'}{2} = \frac{\gamma}{\rho v}$	$\frac{1}{We} = \frac{E}{\rho v^2 d^2}$

number of which is k. In the case of stationary flow, the following three equations hold good

$$\text{div } [\rho v] - \text{div } [D \text{ grad } c_i] + \omega\beta\Delta c_i \pm v_i r = 0,\ i = 1, 2, \ldots, k \qquad (7)$$

$$\text{div } [\rho c_p Tv] - \text{div } [\lambda \text{ grad } T] + \omega\alpha\Delta T \pm v_i r \Delta H = 0 \qquad (8)$$

$$\text{Div } \{\rho v_0 v\} - \text{Div } \{\eta \text{ Grad } v\} + \omega\gamma\Delta v \pm \text{grad } p = 0. \qquad (9)$$

Among them the first two equations are the so called enlarged Damköhler [7] equations,* whereas the third one is a somewhat modified form of the Navier–Stokes equation [8], well known in fluid mechanics. In every equation the first term means *convection*, the second *conduction* (diffusion), the third *transfer between two phases* and the fourth a *source* (respectively a sink, with a negative sign). The above equations are of course homogeneous dimensionally, i.e. the dimension of each term is the same, and it has the same unit of measurement (quantity/m³h), within one unit system. If not the values of the individual terms, but their values related to any selected term are considered, dimensionless quantities will be obtained. Let us write these quantities, e.g. dividing them by a convective term (see Table 1). In the first case its reciprocal is taken.

Note: the symbols of the individual quantities are shown in the table at the end of this paper. It is to be mentioned also, that in the case of momentum flow several dimensionless quantities belong to the last column, since E means force in a general sense. With $E = \Delta p d^2$, a compressive force, we get *Eu* (the Euler number), while with $E = \rho g d^3$, gravitational force, *Fa* (the Fanning number) is obtained, etc.

In this way, $3 \times 3 = 9$ independent dimensionless quantities are obtained for the three streams. Other dimensionless quantities may of course be formed from the four members of the equation, but the number of independent quantities forming the basis always equals nine. For example, in the 2nd column of Table 2, *instead of* (transfer/convection) the dimensionless quantities *Nu'*, *Nu*, and *A** (unnamed) may be formed, corresponding to the ratio (transfer/conduction).

With respect to the latter basis, Table 2 shows the system according to van Krevelen [9], including all the dimensionless quantities derivable from equations (7), (8) and (9). It can be seen that his systematization was effected by group theory, since by means of the basic criteria shown in the frame, all the other elements of the group may be obtained according to equation (5), viz.:

* It is called enlarged, since the third members ($\omega\beta\Delta c_i$) and ($\omega\alpha\Delta T$) shown here and covering transfer processes between phases, are not considered in the original equation by Damköhler. In connection with equation (9), it must be emphasized that the sign Div with a capital letter does not represent a simple divergency, because the quantities in the figure bracket are (secondary) tensors: the dyadic product of the convective momentum flux ρv and v vectors, denoted by a small circle. Similarly, in the second term the capital letter "G" expresses the fact that it is not the gradient-vector of a scalar quantity, but the gradient-tensor of a vector field (in our case the velocity field).

Table 2. Systematization of dimensionless numbers according to van Krevelen

	$\frac{\text{convection}}{\text{conduction}}$ (1)	$\frac{\text{transfer}}{\text{conduction}}$ (2)	$\frac{\text{source}}{\text{convection}}$ (3)	(4)	$\frac{\text{transfer}}{\text{convection}}$ (5)	$\frac{\text{source}}{\text{conduction}}$ (6)	$\frac{\text{source}}{\text{transfer}}$ (7)
Component	$Pe' = \frac{vd}{D}$	$Nu' = \frac{\beta d}{D}$	$Da\text{I} = \frac{r'd}{v}$	$Sc = \frac{Pe'}{Re}$	$St' = \frac{\beta}{v}$	$Da\text{II} = \frac{r'd^2}{D}$	$C^* = \frac{r'd}{\beta}$
Heat	$Pe = \frac{vd}{a}$	$Nu = \frac{\alpha d}{\lambda}$	$Da\text{III} = \frac{\Delta Hrd}{\rho c_p \Delta Tv}$	$Pr = \frac{Pe}{Re}$	$St = \frac{\alpha}{\rho c_p v}$	$Da\text{IV} = \frac{\Delta Hrd^2}{\lambda \Delta T}$	$D^* = \frac{\Delta Hrd}{\alpha \Delta T}$
Momentum	$Re = \frac{vd}{\nu}$	$A^* = \frac{\gamma d}{\eta}$	$\frac{1}{We} = \frac{E}{\rho v^2 d^2}$	$Le = \frac{Sc}{Pr}$	$\frac{f'}{2} = \frac{\gamma}{\rho v}$	$B^* = \frac{E}{\eta v d}$	$E^* = \frac{E}{\gamma d^2 \Delta v}$

$$St' = Nu'\, Pe'^{-1}$$

$$St = Nu\, Pe^{-1}$$

$$\frac{f'}{2} = A^*\, Re^{-1}$$

$$Da\text{II} = Pe'\; Da\text{I}$$

$$Da\text{IV} = Pe\; Da\text{III}$$

$$B^* = Re\; We^{-1}$$

$$C^* = Pe'\,(Nu')^{-1}\; Da\text{I}$$

$$D^* = Pe\; Nu^{-1}\; Da\text{III}$$

$$E^* = Re\,(A^*)^{-1}\; We^{-1}. \qquad (10)$$

It is a great advantage of the systematization by group theory that the determination of the question, *how many* basic criteria there are and *which*: may these be, is made according to exact instructions. As a matter of fact, one group may have *several* bases. In a physical sense it means that the variables of a system may be obtained by different basic criteria having always the same number. In other words, the basic criteria are interchangeable. In the course of the calculations, such changes have already been carried out, but not always in the right way, as will be shown in the following. The right answer is given by the group theory and any arbitrary proceedings are eliminated by it.

According to relation (5), an arbitrary $\bar{B}_i$ element of the system is now written in the form of a power product, providing the whole exponents with double subscripts, where the first subscript denotes the basis, and the second subscript is the subscript of the series:

$$\bar{B}_i = B_1^{a_{i1}} B_2^{a_{i2}} \ldots B_p^{a_{ip}}. \qquad (11)$$

The theorem applying to the selection of the basic elements is:

Should $(B_1, B_2 \ldots B_p)$ be the basic variables (basic elements) of the system, other $(\bar{B}_1, \bar{B}_2 \ldots \bar{B}_p)$ elements, the number of which is equally p, will be basic variables only in the case if any basic element B_j of the previous basic system may be obtained from them with different whole exponents, making

$$B_j = \bar{B}_1^{b_{j1}} \bar{B}_2^{b_{j2}} \ldots \bar{B}_p^{b_{jp}}. \qquad (12)$$

The necessary and sufficient condition for this is, that the determinant formed from both exponent systems should fulfil the requirement of the linear independence, i.e. its value should amount to ± 1.

$$\text{Det} \mid a_{ik} \mid = \text{Det} \mid b_{jk} \mid = \pm 1. \qquad (13)$$

Therefore the elements of a basic system may be exchanged for other elements only then, if the new system thus obtained fulfils the restrictive equation (13). This means that the basic systems are *equivalent* among them. Any element of the group must be obtainable by any basic system.

To illustrate this theorem, it will be shown that the change effected in the second column of the basis of Table 1 and Table 2 is permitted. The determinant formed from the exponents of the two bases may be obtained by employing the first three correlations of (10), in accordance with the restrictive equation (13) covering the condition of linear independence.

	Pe'	Pe	Re	Nu'	Nu	A^*	DaI	DaIII	We^{-1}
Pe'	1	0	0	0	0	0	0	0	0
Pe	0	1	0	0	0	0	0	0	0
Re	0	0	1	0	0	0	0	0	0
St'	−1	0	0	1	0	0	0	0	0
St	0	−1	0	0	1	0	0	0	0
$f'/2$	0	0	−1	0	0	1	0	0	0
DaI	0	0	0	0	0	0	1	0	0
DaIII	0	0	0	0	0	0	0	1	0
We^{-1}	0	0	0	0	0	0	0	0	1

The value of the determinant thus produced is 1. Thus after making the change, the nine elements together also form a basis.

By means of the group theory, it can therefore be substantiated why and how the relations expressed by dimensionless variables may be obtained in engineering calculations. For example, why the well known relation

$$St = \frac{Nu}{Re\,Pr} = \frac{Nu}{Pe} \tag{14}$$

is valid. This is a relation according to equation (5). Similarly, the other *fractional exponent* power product relations may be disclosed and we may say how many dimensionless quantities are included into the relation and which are these quantities. The detailed substantiation of this is, however, too far reaching, and would require a separate lecture.

In conclusion, a case should be submitted where the group theoretical treatment gave a new result. This is the determination of the criteria of scaling up. It is known that two systems having different dimensions are *completely similar* to one another if the homogeneous linear relation

$$x' = kx \tag{15}$$

is valid for their independent variables. These variables may be divided into four groups: 1. geometrical, 2. mechanical, 3. thermal, and 4. component determining, i.e. chemical variables. The variables of the latter three groups are covered by equations (7), (8) and (9).

According to this classification, there exist geometrical, mechanical, thermal, and chemical similitudes.

The geometrical similitude requires that the characteristic length (d) should be proportional in the model and the prototype:

$$d_p = K\,d_M. \tag{16}$$

The mechanical similitude is realized when the values of the three (framed) basic variables shown in the last row of Table 2, connected with momentum conservation, are changed proportionally. The three basic variables diminish to *one*, if the pressure drop (Δp) in the system can be neglected, since then the values A^* and We become practically zero. The mechanical similitude then requires that the Re numbers in the two systems should be equal. If the material constants of both systems (α, β, γ, c_p, η, ρ, λ, ν_i, D_j, ΔH) are identical, the identity of the Re number in the model and prototype

$$v_P d_P = v_M d_M \tag{17}$$

is simplified to an equality. For systems without chemical reaction, the thermal and chemical similitude is included into the conditions (16) and (17). It does not mean therefore another restriction between the variables of the model and the prototype, since the basic variables characteristic for the component and heat streams in columns 1 and 2 of Table 2 (Pe', Pe, Nu' and Nu) contain besides the material constants, the values of which are identified in the two systems, only the variables v and d. The situation is considerably more complicated if a chemical reaction also takes place in the system. Apart from the pressure drop, two more basic variables, DaI and DaIII appear. The thermal similitude is realized in this case only if

the values of *Da*III are identical in the two systems. Disregarding the material constants, the two numbers *Da*III are equal, if the equality

$$\left(\frac{rd}{v\Delta T}\right)_P = \left(\frac{rd}{v\Delta T}\right)_M. \tag{18}$$

holds good.

The condition of the chemical similitude is the equality of the two *Da*I numbers in the two systems, which is given by equation

$$\left(\frac{rd}{v}\right)_P = \left(\frac{rd}{v}\right)_M. \tag{19}$$

The four characteristic variables and the four restriction equations of the four similitudes are compiled in Table 3.

Table 3

Similitude	Characteristic variable	Restricting equation
Geometrical	d	$d_P = Kd_M$
Mechanical	v	$v_P d_P = v_M d_M$
Thermal	ΔT	$\left(\frac{rd}{v\Delta T}\right)_P = \left(\frac{rd}{v\Delta T}\right)_M$
Chemical	r	$\left(\frac{rd}{v}\right)_P = \left(\frac{rd}{v}\right)_M$

The similitude has therefore no degree of freedom, since there are four variables and four equations. This means that if we want to produce a system similar to a given system, none of the variables may be selected freely, but all the variables of the original system must be transformed according to relation (15), i.e. corresponding to exact prescriptions.

On the basis of Damköhler's original work [7], a statement has been spread in the scientific literature, viz. that at scaling up the criterion of complete similitude is the identity of the four *Da* numbers and that of the *Re* numbers in the two systems [10]. As a result of the group theory investigations, this opinion must be corrected, and favourably, too, since *fewer* criteria have to be taken into account. It is correct to say that the criteria of complete similitude between two systems will be according to (16), (17), (18) and (19).

The proportionality of one characteristic dimension, the identity of the *Re* number, the identity of the *Da*I number, and the identity of the *Da*III number. As it has been disclosed, only the *Da*I and *Da*III numbers together belong to one basic system, the numbers *Da*II and *Da*IV can only be written instead of the former numbers into the similitude condition, but not together with the *Da*I and *Da*III numbers.

In the opinion of the author, this new systematization by group theory renders the correlations between dimensionless quantities *simpler* and clearer.

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Résumé—On sait que les grandeurs physiques forment au sens de l'algèbre un groupe abélien libre infini. On montre dans cet article que les quantités sans dimensions d'un système donné forme un groupe abélien libre fini. Il s'ensuit que tout élément du groupe peut être obtenu sous la forme d'un produit de puissances entières, les éléments de ce produit de puissances sont appelés éléments de base. Une nouvelle systématisation logique des quantités sans dimensions par la théorie des groupes est possible.

Les résultats principaux de cette systématisation sont les suivants.

1° Le nombre d'éléments de base est le même que celui des degrés de liberté.

2° On a éliminé tout processus arbitraire de détermination des quantités de base sans dimensions et on

a déterminé en accord à des instructions exactes la question suivante: combien y a-t-il de critères de base et que peuvent-ils être?

3° Il est possible d'expliquer les relations entre les quantités sans dimensions. Par exemple, il peut être déterminé combien de quantités sans dimensions sont en puissance dans les relations et quelles sont ces quantités.

On a donné des exemples de détermination de critères de similitude. On a discuté les équations de base du génie chimique.

Zusammenfassung—Es ist bekannt, dass physikalische Grössen im algebraischen Sinn eine unendlich, freie Abelsche Gruppe bilden. In der Arbeit wird gezeigt, dass die dimensionslosen Zahlen für ein gegebenes System eine freie Abelsche Gruppe ergeben. Daraus folgt, dass jedes Element der Gruppe als Potenzprodukt mit ganzzahligen Exponenten erhalten werden kann. Die Glieder dieser Potenzprodukte werden Grundelemente genannt. Eine neue logische Systematisierung der dimensionslosen Zahlen wurde durch die Gruppentheorie ermöglicht.
Die Hauptergebnisse dieser Systematisierung sind folgende:

1. Die Zahl der Grundelemente ist identisch mit den Freiheitsgraden.
2. Jedes willkürliche Vorgehen bei der Bestimmung der dimensionslosen Grundzahlen wird ausgeschaltet, da die Fragen, welche und wieviele Grundkriterien vorhanden sind, nach genauen Anweisungen beantwortet werden können.
3. Es ist möglich, die Beziehungen zwischen den dimensionslosen Zahlen zu erklären, da bestimmt werden kann, welche und wieviele dimensionslose Zahlen von der Beziehung eingeschlossen werden.

Beispiele zur Bestimmung von Massstabsvergrösserungen sind angegeben. Die Grundgleichungen des Chimie-Ingenieurwesens werden diskutiert.

Аннотация—Как известно, физические величины алгебраически образуют бесконечную свободную абелеву группу. В настоящей статье показано, что безразмерные величины данной системы образуют конечную свободную абелеву группу. Из этого факта вытекает, что любой элемент данной группы можно получить в виде произведения степеней других элементов с целыми показателями. Сомножители, входящие в эти степенные произведения, называются базисными элементами. На основе теории групп возможна новая логическая систематизация безразмерных величин.
Из этой систематизации вытекают следующие основные результаты:

1. Число базисных элементов совпадает с числом степеней свободы.
2. Какое-либо произвольное определение базисных величин исключается, так как решение вопроса о том, какие критерии являются базисными и сколько их, выполняется по точным правилам.
3. Возможно объяснение соотношения между безразмерными величинами, а именно: можно определить, какие и сколько безразмерных величин включается в данное соотношение.

Приводятся примеры представления критериев друг через друга. Обсуждаются основные уравнения химической технологии.